UPPER SEMICONTINUITY AND CLOSEDNESS OF THE SOLUTION SETS TO PARAMETRIC QUASIEQUILIBRIUM PROBLEMS

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ABSTRACT

In this paper we establish sufficient conditions for the solution mappings of parametric generalized vector quasiequilibrium problems to have the stability properties such as upper semicontinuity and closedness. Our results improve recent existing ones in the literature.

Keywords: parametric quasiequilibrium problems, upper semicontinuity, closedness.

TÓM TẮT

Tính chất nửa liên tục trên và tính đóng của các tập nghiệm của các bài toán tựa cân bằng tổng quát phụ thuộc tham số

Trong bài báo này, chúng tôi thiết lập điều kiện đủ cho các tập nghiệm của các bài toán tựa cân bằng tổng quát phụ thuộc tham số có các tính chất ổn định như: tính nửa liên tục trên và tính đóng. Kết quả của chúng tôi là cải thiện một số kết quả tồn tại gần đây trong danh sách tài liệu tham khảo.

Từ khóa: các bài toán tựa cân bằng tổng quát phụ thuộc tham số, tính nửa liên tục trên, tính đóng.

1. Introduction and Preliminaries

Let X, Y, Λ, Γ, M be Hausdorff topological spaces, let Z be a Hausdorff topological vector space, $A \subseteq X$ and $B \subseteq Y$ be nonempty sets. Let $K_1 : A \times \Lambda \to 2^A$, $K_2 : A \times \Lambda \to 2^A$, $T : A \times A \times \Gamma \to 2^B$, $C : A \times \Lambda \to 2^B$ and $F : A \times B \times A \times M \to 2^Z$ be multifunctions with $C(x,\lambda)$ is a proper convex cone values and closed.

Now, we adopt the following notations. Letters w, m and s are used for a weak, middle and strong, respectively, kinds of considered problems. For subsets U and V under consideration we adopt the notations.

(u,v) w $U \times V$ means $\forall u \in U, \exists v \in V$,

(u,v) m $U \times V$ means $\exists v \in V, \forall u \in U$,

(u,v) s $U \times V$ means $\forall u \in U, \forall v \in V$,

(u,v) $\overline{w} U \times V$ means $\exists u \in U, \forall v \in V$ and similarly for $\overline{m}, \overline{s}$.

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Let $\alpha \in \{\mathrm{w, m, s}\}$ and $\bar{\alpha} \in \{\bar{w}, \bar{m}, \bar{s}\}$. We consider the following parametric quasiequilibrium problem (in short, (QEP$_{\alpha}^{\lambda\gamma\mu}$)):

(QEP$_{\alpha}^{\lambda\gamma\mu}$): Find $\bar{x} \in K_1(\bar{x}, \lambda)$ such that $(y,t)\alpha K_2(\bar{x}, \lambda) \times T(\bar{x}, y, \gamma)$ statisfying

$F(\bar{x}, t, y, \mu) \not\subseteq -\mathrm{intC}(\bar{x}, \lambda)$.

For $\lambda \in \Lambda, \gamma \in \Gamma, \mu \in M$ consider the following parametric extended quasiequilibrium problem (in short, (QEEP$_{\alpha}^{\lambda\gamma\mu}$)):

(QEEP$_{\alpha}^{\lambda\gamma\mu}$): Find $\bar{x} \in K_1(\bar{x}, \lambda)$ such that $(y,t)\alpha K_2(\bar{x}, \lambda) \times T(\bar{x}, y, \gamma)$ statisfying

$F(\bar{x}, t, y, \mu) \cap -\mathrm{intC}(\bar{\mathrm{x}}, \lambda) = \varnothing$.

For each $\lambda \in \Lambda, \gamma \in \Gamma, \mu \in M$, we let $E(\lambda) := \{x \in A \mid x \in K_1(x, \lambda)\}$ and let $\Sigma_{\alpha}, \Xi_{\alpha} : \Lambda \times \Gamma \times M \to 2^A$ be set-valued mappings such that $\Sigma_{\alpha}(\lambda, \gamma, \mu)$ and $\Xi_{\alpha}(\lambda, \gamma, \mu)$ are the solution sets of (QEP$_{\alpha}^{\lambda\gamma\mu}$) and (QEEP$_{\alpha}^{\lambda\gamma\mu}$), respectively.

Throughout the paper we assume that $\Sigma_{\alpha}(\lambda, \gamma, \mu) \neq \varnothing$ and $\Xi_{\alpha}(\lambda, \gamma, \mu) \neq \varnothing$ for each (λ, γ, μ) in the neighborhoods $(\lambda_0, \gamma_0, \mu_0) \in \Lambda \times \Gamma \times M$.

By the definition, the following relations are clear:

$\Sigma_{\mathrm{s}} \subseteq \Sigma_{\mathrm{m}} \subseteq \Sigma_{\mathrm{w}}$ and $\Xi_{\mathrm{s}} \subseteq \Xi_{\mathrm{m}} \subseteq \Xi_{\mathrm{w}}$.

Special cases of the problems (QEP$_{\alpha}^{\lambda\gamma\mu}$) and (QEEP$_{\alpha}^{\lambda\gamma\mu}$) are as follows:

(a) If $T(x,y,\gamma) = \{t\}, \Lambda = \Gamma = M, A = B, X = Y, K_1 = K_2 = K$ and $\alpha = m$, then (QEP$_{\alpha}^{\lambda\gamma\mu}$) and (QEEP$_{\alpha}^{\lambda\gamma\mu}$) become to (PGQVEP) and (PEQVEP), respectively in Kimura-Yao [8]

(PGQVEP): Find $\bar{x} \in K(\bar{x}, \lambda)$ such that

$F(\bar{x}, y, \lambda) \not\subset -\operatorname{int} C(\bar{x}, \lambda))$, for all $y \in K(\bar{x}, \lambda)$.

and

(PEQVEP): Find $\bar{x} \in K(\bar{x}, \lambda)$ such that

$F(\bar{x}, y, \lambda) \cap -\operatorname{int} C(\bar{x}, \lambda) = \varnothing$, for all $y \in K(\bar{x}, \lambda)$.

(b) If $T(x,y,\gamma) = \{t\}, \Lambda = \Gamma, A = B, X = Y, K_1 = clK, K_2 = K, \alpha = m, C(x,\lambda) \equiv C$ and replace "$\not\subseteq -\operatorname{int} C(x,\lambda)$" by "$\subseteq Z$, $-\operatorname{int} C$" with $C \subseteq Z$ be closed and $\operatorname{int} C \neq \varnothing$, then (QEP$_{\alpha}^{\lambda\gamma\mu}$) become to (SQEP) in Anh-Khanh [1].

(SQEP): Find $\bar{x} \in K(\bar{x}, \lambda)$ such that

$F(\bar{x}, y, \lambda) \subseteq Z$, $-\operatorname{int} C$, for all $y \in K(\bar{x}, \lambda)$.

(c) If $T(x,y,\gamma) = \{t\}, \Lambda = \Gamma = M, A = B, X = Y, K_1 = K_2 = K, \alpha = m$ and replace F by f be a vector function, then (QEP$_{\alpha}^{\lambda\gamma\mu}$) become to (PVQEP) in Kimura-Yao [7].

(PQVEP): Find $\bar{x} \in K(\bar{x}, \lambda)$ such that

$f(\bar{x}, y, \lambda) \notin -\operatorname{int} C(\bar{x}, \lambda))$, for all $y \in K(x, \lambda)$.

The parametric generalized quasiequilibrium problems include many rather general problems as particular cases as vector minimization, variational inequalities, Nash equilibria, fixedpoint and coincidence-point problems, complementarity problems, minimax inequalities, etc. Stability properties of solutions have been investigated even in models for vector quasiequilibrium problems [1, 3, 4, 7, 8, 9], variational problems [5, 6, 10, 11] and the references therein.

In this paper we establish sufficient conditions for the solution sets $\Sigma_\alpha, \Xi_\alpha$ to have the stability properties such as the upper semicontinuity and closedness with respect to parameter λ, γ, μ.

The structure of our paper is as follows. In the remaining part of this section we recall definitions for later uses. Section 2 is devoted to the upper semicontinuity and closedness of solution sets for parametric quasiequilibrium problems (QEP$_\alpha^{\lambda\gamma\mu}$) and (QEEP$_\alpha^{\lambda\gamma\mu}$).

Now we recall some notions in [1, 2, 12]. Let X and Z be as above and $G: X \to 2^Z$ be a multifunction. G is said to be lower semicontinuous (lsc) at x_0 if $G(x_0) \cap U \neq \emptyset$ for some open set $U \subseteq Z$ implies the existence of a neighborhood N of x_0 such that, for all $x \in N, G(x) \cap U \neq \emptyset$. An equivalent formulation is that: G is lsc at x_0 if $\forall x_\alpha \to x_0$, $\forall z_0 \in G(x_0), \exists z_\alpha \in G(x_\alpha), z_\alpha \to z_0$. G is called upper semicontinuous (usc) at x_0 if for each open set $U \supseteq G(x_0)$, there is a neighborhood N of x_0 such that $U \supseteq G(N)$. G is said to be Hausdorff upper semicontinuous (H-usc in short; Hausdorff lower semicontinuous, H-lsc, respectively) at x_0 if for each neighborhood B of the origin in Z, there exists a neighborhood N of x_0 such that, $G(x) \subseteq G(x_0) + B, \forall x \in N$ ($G(x_0) \subseteq G(x) + B, \forall x \in N$). G is said to be continuous at x_0 if it is both lsc and usc at x_0 and to be H-continuous at x_0 if it is both H-lsc and H-usc at x_0. We say that G satisfies a certain property in a subset $A \subseteq X$ if G satisfies it at all points of A.

Proposition 1.1. *(See [1, 2, 12]) Let A and Z be as above and $G: A \to 2^Z$ be a multifunction.*

(i) If G is usc at x_0 then G is H-usc at x_0. Conversely if G is H-usc at x_0 and if $G(x_0)$ compact, then G is usc at x_0;

(ii If G is usc at x_0 and if $G(x_0)$ is closed, then G is closed at x_0;

(iii) If Z is compact and G is closed at x_0 then G is usc at x_0;

(iv) If G has compact values, then G is usc at x_0 if and only if, for each net $\{x_\alpha\} \subseteq A$ which converges to x_0 and for each net $\{y_\alpha\} \subseteq G(x_\alpha)$, there are $y \in G(x_0)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \to y$.

2. Main results

In this section, we discuss the upper semicontinuity and closedness of solution sets for parametric quasiequilibrium problems (QEP$_\alpha^{\lambda\gamma\mu}$) and (QEEP$_\alpha^{\lambda\gamma\mu}$).

Theorem 2.1.

Assume for problem (QEP$_\alpha^{\lambda\gamma\mu}$) that

(i) E is usc at λ_0 and $E(\lambda_0)$ is compact, and K_2 is lsc in $K_1(A,\Lambda)\times\{\lambda_0\}$;

(ii) in $K_1(A,\Lambda)\times K_2(K_1(A,\Lambda),\Lambda)\times\{\gamma_0\}$, T is usc and compact-valued if $\alpha = w$ (or $\alpha = m$), and lsc if $\alpha = s$;

(iii) the set $\{(x,t,y,\mu,\lambda) \in K_1(A,\Lambda)\times T(K_1(A,\Lambda),K_2(K_1(A,\Lambda),\Lambda),\Gamma)\times$

$K_2(K_1(A,\Lambda),\Lambda)\times\{\mu_0\}\times\{\lambda_0\} : F(x,t,y,\mu) \not\subseteq -\operatorname{int}C(x,\lambda)\}$ is closed.

Then Σ_α is both upper semicontinuous and closed at $(\lambda_0,\gamma_0,\mu_0)$.

Proof. Similar arguments can be applied to three cases. We present only the proof for the cases where $\alpha = w$. We first prove that Σ_w is upper semicontinuous at $(\lambda_0,\gamma_0,\mu_0)$.

Indeed, we suppose to the contrary that Σ_w is not upper semicontinuous at $(\lambda_0,\gamma_0,\mu_0)$, i.e., there is an open set U of $\Sigma_w(\lambda_0,\gamma_0,\mu_0)$ such that for all $\{(\lambda_n,\gamma_n,\mu_n)\}$ convergent to $\{(\lambda_0,\gamma_0,\mu_0)\}$, there exists $x_n \in \Sigma_w(\lambda_n,\gamma_n,\mu_n)$, $x_n \notin U$, $\forall n$. By the upper semicontinuity of E and compactness of $E(\lambda_0)$, one can assume that $x_n \to x_0$ for some $x_0 \in E(\lambda_0)$. If $x_0 \notin \Sigma_w(\lambda_0,\gamma_0,\mu_0)$, then $\exists y_0 \in K_2(x_0,\lambda_0), \forall t_0 \in T(x_0,y_0,\gamma_0)$ such that

$$F(x_0,t_0,y_0,\mu_0) \subseteq -\operatorname{int}C(x_0,\lambda_0). \quad (2.1)$$

By the lower semicontinuity of K_2 at (x_0,λ_0), there exists $y_n \in K_2(x_n,\lambda_n)$ such that $y_n \to y_0$. Since $x_n \in \Sigma_w(\lambda_n,\gamma_n,\mu_n)$, $\exists t_n \in T(x_n,y_n,\gamma_n)$ such that

$$F(x_n,t_n,y_n,\mu_n) \not\subseteq -\operatorname{int}C(x_n,\lambda_n). \quad (2.2)$$

Since T is usc and $T(x_0,y_0,\gamma_0)$ is compact, one has a subnet $t_m \in T(x_m,y_m,\gamma_m)$ such that $t_m \to t_0$ for some $t_0 \in T(x_0,y_0,\gamma_0)$.

By the condition (iii) we see a contradiction between (2.1) and (2.2). Thus, $x_0 \in \Sigma_w(\lambda_0,\gamma_0,\mu_0) \subseteq U$, this contradicts to the fact $x_n \notin U$, $\forall n$. Hence, Σ_w is upper semicontinuous at $(\lambda_0,\gamma_0,\mu_0)$.

Now we prove that Σ_w is closed at $(\lambda_0,\gamma_0,\mu_0)$. Indeed, we suppose that Σ_w is not closed at $(\lambda_0,\gamma_0,\mu_0)$, i.e., there is a net $(x_n,\lambda_n,\gamma_n,\mu_n)\to(x_0,\lambda_0,\gamma_0,\mu_0)$ with $x_n\in\Sigma_w(\lambda_n,\gamma_n,\mu_n)$ but $x_0\notin\Sigma_w(\lambda_0,\gamma_0,\mu_0)$. The further argument is the same as above. And so we have Σ_w is closed at $(\lambda_0,\gamma_0,\mu_0)$. □

The following example shows that the upper semicontinuity and compactness of E are essential.

Example 2.2.

Let $A=B=X=Y=\mathbb{R}$, $\Lambda=\Gamma=M=[0,1],\ \lambda_0=0, C(x,\lambda)=\mathbb{R}$,

$F(x,t,y,\lambda)=3^{2\lambda+\sin x}, K_1(x,\lambda)=(-\lambda-1,\lambda], K_2(x,\lambda)=\{0\}$ and $T(x,y,\lambda)=[0,2^{3^x+2\cos\lambda}]$.

Then, we have $E(0)=(-1,0]$ and $E(\lambda)=(-\lambda-1,\lambda],\forall\lambda\in(0,1]$. We show that K_2 is lsc and assumption (ii) and (iii) of Theorem 2.1 are fulfilled. But Σ_α is neither usc nor closed at $\lambda_0=0$ and $\Sigma_\alpha(0,0,0)$ is not compact. The reason is that E is not usc at 0 and $E(0)$ is not compact. In fact $\Sigma_\alpha(0,0,0)=(-1,0]$ and $\Sigma_\alpha(\lambda,\gamma,\mu)=(-\lambda-1,\lambda],\forall\lambda\in(0,1]$.

Remark 2.3.

The assumption in Theorem 2.1 we have K_2 is lsc in $K_1(A,\Lambda)\times\{\lambda_0\}$ (which is not imposed in this Theorem 4.1 of [8] and [7]). Example 2.4 shows that the lower semicontinuity of K_2 needs to be added to Theorem 4.1 of [8] and [7].

Example 2.4.

Let $X,Y,\Lambda,\Gamma,M,\lambda_0,C(x,\lambda)$ as in Example 2.2 and let $A=B=[-\frac{1}{2},\frac{1}{2}]$,

$F(x,t,y,\lambda)=x+y+\lambda, K_1(x,\lambda)=[0,\frac{1}{2}], T(x,y,\lambda)=\{t\}$. We have

$$K_2(x,\lambda)=\begin{cases}\left\{-\frac{1}{2},0,\frac{1}{2}\right\} & \text{if } \lambda=0,\\ \left\{0,\frac{1}{2}\right\} & \textit{otherwise.}\end{cases}$$

We have $E(\lambda)=[0,1],\forall\lambda\in[0,1]$. Hence E is usc at 0 and $E(0)$ is compact and condition (ii) and (iii) of Theorem 2.1 are easily seen to be fulfilled.

But Σ_α is not upper semicontinuous at $\lambda_0=0$. The reason is that K_2 is not lower semicontinuous. In fact

$$\Sigma_\alpha(\lambda,\gamma,\mu) == \begin{cases} \left\{\frac{1}{2}\right\} & \text{if } \lambda = 0, \\ \left[0,\frac{1}{2}\right] & otherwise. \end{cases}$$

The following example shows that the condition (iii) of Theorem 2.1 is essential.

Example 2.5.

Let $\Lambda,\Gamma,M,T,\lambda_0,C$ as in Example 2.4 and let $X = Y = A = B = [0,1]$,

$K_1(x,\lambda) = K_2(x,\lambda) = [0,1]$ and

$$F(x,t,y,\lambda) = \begin{cases} \frac{x-y}{2} & \text{if } \lambda = 0, \\ \frac{y}{3} - \frac{x}{2} & otherwise. \end{cases}$$

We show that assumptions (i) and (ii) of Theorem 2.1 are easily seen to be fulfilled.

But Σ_α is not usc at $\lambda_0 = 0$. The reason is that assumption (iii) is violated.

Indeed, taking $x_n = 0, t_n = 0, y_n = \frac{1}{2}, \lambda_n = \frac{1}{n} \to 0$ as $n \to \infty$, then $\{(x_n, y_n, \lambda_n)\} \to (0,\frac{1}{2},0)$ and $F(x_n,t_n,y_n,\lambda_n) = F(0,0,\frac{1}{2},1/n) = \frac{1}{6} > 0$, but $F(0,0,1,0) = -\frac{1}{4} < 0$.

The following example shows that all assumptions of Theorem 2.1 are fulfilled. But Theorem 3.4 in Anh and Khanh [1] cannot be applied.

Example 2.6.

Let $A,B,X,Y,\Lambda,\Gamma,M,\lambda_0,C$ as in Example2.5 and let $K_1(x,\lambda) = K_2(x,\lambda) = [0,2], T(x,y,\gamma) = [0,1]$

$$F(x,t,y,\lambda) = \begin{cases} 0 & \text{if } \lambda = 0, \\ e^{\cos^2 x + 2} & otherwise. \end{cases}$$

We show that assumptions (i), (ii) and (iii) of Theorem 2.1 are easily seen to be fulfilled. Hence, Σ_α is usc at $(0,0,0)$. But Theorem 3.4 in Anh and Khanh [1] cannot be applied. The reason is that F is not lsc at $(x,y,0)$.

Remark 2.7.

(i) In Theorem 4.1 in Kimura-Yao [8] the same conclusion as Theorem 2.1 was proved in another way. Its assumptions (i)-(iv) derive (i) Theorem 2.1, assumptions (v)(or (vi)) coincides with (iii) of Theorem 2.1.

(ii) *In* Theorem 4.1 in Kimura-Yao [7] the same conclusion as Theorem 2.1 was proved in another way. Its assumptions (i)-(iv) derive (i) Theorem 2.1, assumption (v) coincides with (iii) of Theorem 2.1.

Theorem 2.8.

Assume for problem $(QEEP_{\alpha}^{\lambda\gamma\mu})$ *that*

(i) E *is usc at* λ_0 *and* $E(\lambda_0)$ *is compact, and* K_2 *is lsc in* $K_1(A,\Lambda)\times\{\lambda_0\}$*;*

(ii) in $K_1(A,\Lambda)\times K_2(K_1(A,\Lambda),\Lambda)\times\{\gamma_0\}$*,* T *is usc and compact-valued if* $\alpha = w$ *(or* $\alpha = m$*), and lsc if* $\alpha = s$*;*

(iii) the set $\{(x,t,y,\mu,\lambda)\in K_1(A,\Lambda)\times T(K_1(A,\Lambda),K_2(K_1(A,\Lambda),\Lambda),\Gamma)\times K_2(K_1(A,\Lambda),\Lambda)\times\{\mu_0\}\times\{\lambda_0\}: F(x,t,y,\mu)\cap -\operatorname{int} C(x,\lambda)=\varnothing\}$ *is closed.*

Then Ξ_α *is both upper semicontinuous and closed at* $(\lambda_0,\gamma_0,\mu_0)$.

Proof. Similar arguments can be applied to three cases. We present only the proof for the cases where $\alpha = m$. We first prove that Ξ_m is upper semicontinuous at $(\lambda_0,\gamma_0,\mu_0)$. Indeed, we suppose to the contrary that Ξ_m is not upper semicontinuous at $(\lambda_0,\gamma_0,\mu_0)$, i.e., there is an open set V of $\Xi_m(\lambda_0,\gamma_0,\mu_0)$ such that for all $\{(\lambda_n,\gamma_n,\mu_n)\}$ convergent to $\{(\lambda_0,\gamma_0,\mu_0)\}$, there exists $x_n\in\Xi_m(\lambda_n,\gamma_n,\mu_n)$, $x_n\notin V$, $\forall n$. By the upper semicontinuity of E and compactness of $E(\lambda_0)$, one can assume that $x_n\to x_0$ for some $x_0\in E(\lambda_0)$. If $x_0\notin\Xi_m(\lambda_0,\gamma_0,\mu_0)$, then $\forall t_0\in T(x_0,y_0,\gamma_0),\exists y_0\in K_2(x_0,\lambda_0)$ such that

$$F(x_0,t_0,y_0,\mu_0)\cap -\operatorname{int} C(x_0,\lambda_0)\neq\varnothing. \quad (2.3)$$

By the lower semicontinuity of K_2 at (x_0,λ_0), there exists $y_n\in K_2(x_n,\lambda_n)$ such that $y_n\to y_0$. Since $x_n\in\Xi_m(\lambda_n,\gamma_n,\mu_n)$, $\exists t_n\in T(x_n,y_n,\gamma_n)$ such that

$$F(x_n,t_n,y_n,\mu_n)\cap -\operatorname{int} C(x_n,\lambda_n)=\varnothing. \quad (2.4)$$

Since T is usc and $T(x_0,y_0,\gamma_0)$ is compact, one has a subnet $t_m\in T(x_m,y_m,\gamma_m)$ such that $t_m\to t_0$ for some $t_0\in T(x_0,y_0,\gamma_0)$.

By the condition (iii) we see a contradiction between (2.3) and (2.4). Thus, $x_0\in\Xi_m(\lambda_0,\gamma_0,\mu_0)\subseteq V$, this contradicts to the fact $x_n\notin V$, $\forall n$. Hence, Ξ_m is upper semicontinuous at $(\lambda_0,\gamma_0,\mu_0)$. Now we prove that Ξ_m is closed at $(\lambda_0,\gamma_0,\mu_0)$. Indeed, we suppose that Ξ_m is not closed at $(\lambda_0, \gamma_0, \mu_0)$, i.e., there is a net $(x_n, \lambda_n, \gamma_n, \mu_n) \to (x_0, \lambda_0, \gamma_0, \mu_0)$ with $x_n \in \Xi_m(\lambda_n, \gamma_n, \mu_n)$ but $x_0 \notin \Xi_m(\lambda_0, \gamma_0, \mu_0)$. The further argument is the same as above. And so we have Ξ_m is closed at $(\lambda_0, \gamma_0, \mu_0)$. □

Remark 2.9.

Theorem 2.8 is an extension of Theorem 4.1 in [8]. The Example 2.3 is also shows that the lower semicontinuity of K_2 needs to be added to Theorem 4.1 of Kimura-Yao in [8].

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