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RESEARCH ON MISTAKES OF ECONOMICS AND ENGINEERING STUDENTS IN LEARNING THE TOTAL PROBABILITY

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ABSTRACT

Predicting possible mistakes of the learner in accessing a knowledge and determining their causes are necessary in the teaching process. By approaching obstacles in the didactic perspective, this paper presents a study of the economics and engineering students' mistakes at Saigon University in learning total probability rule through a pedagogical experiment.

Keywords: didactic obstacles, mistakes, total probability rule.

TÓM TẮT

Nghiên cứu sai lầm của sinh viên ngành kinh tế và kĩ thuật khi học xác suất đầy đủ

Dự đoán sai lầm của người học trong việc tiếp cận một tri thức và xác định nguồn gốc của các sai lầm đó là cần thiết trong quá trình dạy học. Từ cách tiếp cận chướng ngại didactic, bài báo này trình bày một nghiên cứu về sai lầm của sinh viên khối ngành kinh tế, kĩ thuật của Trường Đại học Sài Gòn trong học tập xác suất đầy đủ thông qua một thực nghiệm sư phạm.

Từ khóa: chướng ngại didactic, sai lầm, xác suất đầy đủ.

1. Problematic

Problems related to total probability rule often make them difficult for students. Normally, one of the signals to identify the problems is that they are conducted through two experiments and the event of the latter is directly related to the first. For more complex problems, to recognise mutually exclusive subsets is one of the main obstacles to solve the problem. By approaching obstacles in the didactic perspective, we conduct a research on students' mistakes analysis through a learning process in total probablility rule.

2. Theoretical framework

2.1. Didactic Obstacle

Obstacle in in the didactic perspective

According to (Annie,B., Claude,C., Chau, L.T.H., Tien,L.V, 2009, p.59) obstacles have the following characteristics:

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- An obstacle is a knowledge or concept. It is not a difficulty or lack of knowledge.
- This knowledge gives the suitable answers in a certain context where we usually experience.
- Beyond the context, it gives wrong answers. It should be a significant change to have a right answer in every context.
- Moreover, knowledge go against its contradiction and its contradicts the establishment of a more perfect knowledge.
- Even if the subject is aware of knowledge which is inaccuracy. The obstacle is still persistent and not this time.

According to G. Brousseau, pedagogic obstacles come from transformation in pedagogy. It seems to depend on the choice of a teaching system.

2.2. Didactic Contract

A didactic contract a set of rules which limit teacher's responsibilities and student's responsibilities for mathematic knowledge which is taught. It is a set of activity rules and conditions which regulate the relationship between teachers and students.

A didactic contract is considered as a method to study students' mistakes and predict the causes of them.

An effective study method of a didactic contract is to create weird situations that could be a context where teachers and students may break the old situations.

3. Teaching the total probability in Saigon university

In Saigon University's curriculum for economics and engineering, Probability Statistics takes an important position and it is taught as module named Probability Statistics A in the period for career basic knowledge. It is a obligatory course for all economics and engineering students. They must take 45 periods of this module in the first year or the second year when they are learning Advanced Mathematic C1 (for economics) or Analytics (for engineering).

In this paper, we consider the subject O as "Total probability rule" and the institution I as "The institution of teaching and learning Probability Statistics for economics and engineering students at SaiGon University". There are two syllabus named "Applied Probability-Statistics" (Dong L.S.,2012) (textbook TB) and "Exercises Applied Probability-Statistics" (Dong L.S.,2012) (workbook WB).

"Total probability rule" is in chapter 1: "Radom event and probability of event" and is in item 4: "Total probability rule, Bayes rule". This content is from page 23 to page 26 in TB and it is taught after they finish knowledge related to classical probability, probability rules (addition rule, conditional probability rule, multiplication probability rule).

To begin with the knowledge of total probability, textbook represents the definition of mutually exclusive events:

"*Definition: A group of events* $A_1, A_2, \ldots, A_n$ $(n \geq 2)$ *in an experiment is mutually exclusive events if it satisfies the following properties:*

i) $A_1 + A_2 + \ldots + A_n = \Omega$

ii) $A_i A_j = \varnothing$." (Dong L.S.,2012, p.23)

Next, textbook (Dong L.S.,2012, p.23) introduce the total probability rule through a theorem:

"*Theorem: If there are an* A *event and mutually exclusive events in an experiment* $A_1, A_2, \ldots, A_n$ *then*

Total probability rule:

$$P(A) = P(A_1)P(A \mid A_1) + P(A_2)P(A \mid A_2) + \cdots + P(A_n)P(A \mid A_n)$$
$$= \sum_{i=1}^{n} P(A_i)P(A \mid A_i)."$$

Next, TB gives 4 examples related to total probability rule to compute probability of an event.

Through 4 examples in TB and 12 examples in WB, they play an important position to determine the compositions of mathematic organization related to total probability rule. Next, we present two examples in TB (Dong L. S., 2012, p.24) related to the total probability calculation task type:

Example 11: There are two boxes. The first box has 10 products, three of which are defective. The second box has 12 products, four of which are defective. A customer pick one of two boxes at random and then pick two products from that box at random to check. If all of non-defective then he/she buys that box. What is the probabilty that the box is bought?

SOLVING: A = "A box is bought" = "Taking 2 non-defective from this box"

A_i = "Taking the i^{th} box", i = 1,2; A_i is mutually exclusive events.

According to the total probability rule: $P(A) = P(A_1)P(A \mid A_1) + P(A_2)P(A \mid A_2)$.

$$P(A) = \frac{1}{2}.\frac{C_7^2}{C_{10}^2} + \frac{1}{2}.\frac{C_8^2}{C_{12}^2} = 0,4455.$$

Examlpe 12: A company has a plan to release new products to the market. The result of previous experiment shows that 40% of new products are successful and 60% of new products are unsuccessful. Before the products are launched, a market research is conducted on the customers' attitude about "like" or "dislike" products. According to experience, 80% of successful new products and 30% of unsuccessful new products are accepted by customers who are interviewed. If we pick a new product at random,what is the probabilty that customers like the product?

SOLVING: A = "A customer likes a product".

A_1 = "take a successful product", A_2 = "take a unsuccessful product". A_1, A_2 is mutually exclusive events. We have:

$$P(A) = P(A_1)P(A \mid A_1) + P(A_2)P(A \mid A_2)$$
$$P(A) = 0,4.0,8 + 0,6.0,3 = 0,5 = 50\%.$$

In the Table 1 below, we present the statistics table of examples and exercises in TB and WB book related to T-task.

Table 1. *Statistics of the number of the examples and exercises in TB and WB*

	Example	Exercise	Total
TB	4	8	12
WB	12	15	27

All of these examples have the same type of T task: "Compute the probability of an event through mutually exclusive events"and the same technique to solve. Next, we will describe in the detail the steps of the technique involved in the T task above.

3.1. A Task type related to the total probability

T task: Compute the probability of an event through mutually exclusive events.

Technique τ :

- Step 1: A is the event that we need to compute the probability.
- Step 2: Indentify mutually exclusive events.

$\{A_1, A_2, \ldots, A_n\}$ $(n \geq 2)$ that satisfies the following properties:

i) $A_1 + A_2 + \ldots + A_n = \Omega$

ii) $A_i A_j = \varnothing$ $(i \neq j)$.

- Step 3: Compute $P(A_i)$ $(i = 1, ..., n)$.
- Step 4: Compute conditional probabilities $P(A \mid A_i)$ $(i = 1, ..., n)$.
- Step 5: Apply the rule: $P(A) = \sum_{i=1}^{n} P(A_i)P(A \mid A_i)$.

Through example 11, TB identifies mutually exclusive events as $\{A_1$:take the first box, A_2: take the second box$\}$ at step 2. We recognize that $P(A_1), P(A_2)$ and $P(A \mid A_1), P(A \mid A_2)$ are not computed at step 3 and step 4 but they are present in step 5.

Therefore, students assume that step 3 and step 4 could be unnecessary to be presented in details. Consequently, mistakes may be take place in these steps. In term of example 11, the probability of choosing the first box or the second box did not mention but at step 5, these probabilities are showed that $P(A_1) = P(A_2) = \frac{1}{2}$. Therefore, if we have two mutually exclusive events $\{A_1, A_2\}$ and the probabilities of components do not mention, students do not consider to how many percentage of A_i 's chance and think that the probabilities of the outcomes are equal ($P(A_1) = P(A_2) = 0,5$).

Through example 12, TB identifies mutually exclusive events as $\{A_1$: take a successful product, A_2: take a unsuccessful product$\}$ at step 2. We recognize that $P(A_i)\ (i=1,2)$ and $P(A|A_1), P(A|A_2)$ are mentioned respectively, $P(A_1)=40\%$, $P(A_2)=60\%$, $P(A|A_1)=80\%$ and $P(A|A_1)=30\%$. In term of this example, there is not any problem at step 3,4 and 5. However, all of the examples and exercises in both TB and WB have the same property for the percentage of mutually exclusive events. They either are given the percentage or are calculated by some simple steps. Therefore, students think that mutually exclusive events will be chosen so that total percentage of componant events is 100%. It may lead to mistakes when there are many different ways to partition the sample space. Among them, students need to choose one suitable. Hence, if a choice is wrong students cannot calculate the probability of the event.

3.2. Some mistakes related to the T-task

- **Mistake 1: An unsuitable partitioning the sample space Ω**

We are aware that the total probability are computed by mutually exclusive subsets. Thus, if partitioning a sample space into several mutually exclusive subsets is not logical, it leads that we cannot calculate the probability.

We need to analyze the reasons which lead to choose wrong mutually exclusive subsets. They related to didactic obstacle which is generated by the expression of two syllabus. There is an implicit convention in choosing mutually exclusive subsets. They either are given the percentage (%) or are calculated by some simple steps. The implicit convention is didactic contract **R.** Hence, if mutually exclusive events are not represented, it can break the didactic contract.

- **Mistake 2: Mutually exclusive events have the same chance**

According to technique τ, students just use the information of the problem to compute $P(A_i), P(A|A_i)\ (i=1,...,n)$ at step 4 and 5 but they are embarrassed and make mistakes. This is becasue that they think all of these probabilities are equal to $\frac{1}{n}$.

Especially, if we have two mutually exclusive events $\{A_1, A_2\}$, students are not consider to how many percentage of A_i's chance and think that the probabilities of the outcomes are equal to $P(A_1) = P(A_2) = 0,5$. This error related to didactic obstacle in teaching probability. This obstacle has been researched and published in the article by Le Thi Hoai Chau (2010)(Chau, Difficulties and obstacles in teaching probability concepts, 2010). Le Thi Hoai Chau analyzed these obstacle involved in students' viewpoints when they think that each event always has the half of chance to present.

- **Mistake 3: Addition the probability of two events in two different sample space**

The old knowledge such as addition probability rule for two mutually exclusive events affects students. It can lead to make a mistake. In the addition rule section, all of exercises and examples in both TB and WB illustrate that all of mutually exclusive events are generated at the same experiment; therefore, they have the same sample space. Thus, when students learn total probability, the requested event belongs to mutually exclusive subsets. These subsets create several different sub sample space. This is a situation lead to mistakes which are affected by the old knowledge. A table below shows the number of examples and exercises in both TB and WB involved in addition rule of mutually exclusive events in the same experiment.

Table 2. *Statistics of the number of the examples and exercises in SB and WB*

	Example	Exercise	Total
SB	1	4	5
WB	11	4	15

Research hypothesis

The above analysis allowed us to formulate a hypothesis:

Hypothesis ***H****: "There are some mistakes (Mistake 1, Mistake 2, Mistake 3) of students in studying T-task related to total probability".*

4. Test the hypothesis through an experimental research

4.1. Experiment illustration

The cases below are done to test the hypothesis H above. In these cases, we conduct experiment on 58 economics and engineering students at Saigon University after learning knowledge about total probability.

To test three mistakes above we give them two problems and they must finish in 45 minutes.

Students do two problems on A4 paper, including drafts and written assignments. They are collected after completing.

Two problems are given to economics and engineering students after they study classical probability, addition rule, conditional probability, multiplication rule and total probability. Two problems are suitable for the Probability statistics A module which is teaching by us. They involve in basic socio-economic knowledge. To calculate the requested probabilities in the problems, students need to apply the above knowledge especially total probability rule.

The main purpose of problem 1 is to help us verify the presence of Mistake 1 and Mistake 2. The main purpose of problem 2 is to help us verify the presence of Mistake 2 and Mistake 3.

- **Problem 1:** According to a statistic source, the proportion of twins in residential area shows that 34% of twins are boys, 30% of twins are girls, and 36% of twins have different gender. A pair of twins may be born either from the same egg (identical twins) or from two different eggs (non-identical twins). Identical twins are always same gender. On the other hand, non-identical twins are independent in gender and each gender has the same probability equal to 0,5. What is the probability of identical twins?

- **Problem 2:** There are two workshops producing the same type products. The workshop 1 produces 1000 products including 100 defective products. The workshop 2 produces 2000 products including 150 defective products. We pick up one product to test at ramdom. What is the probability of the defective product?

4.2. A prior analysis

a) Problem 1:

- **Possible strategies**

S1: Apply the total probability

In problem 1, there is a partitioning the sample space into two groups such as {twins are boys, twins are girls, twins are the different gender} or {identical twins, non-identical twins}. If students choose the first mutually exclisive events, they cannot calculate the requested probability. The purpose of problem 1 is to help us verify the presence of Mistake 1.

- **Possible techniques**

τ11:

A partitioning the sample space Ω into mutually exclusive events such as {twins are boys, twins are girls, twins are the different gender}

A: "Identical twins".

B: "Twins are boys".

G: "Twins are girls".

D: "Twins are different gender".

$\{B, G, D\}$ are mutually exclusive events.

$$P(A) = P(B)P(A \mid B) + P(G)P(A \mid G) + P(D)P(A \mid D)$$
$$= 0{,}34.0{,}5 + 0{,}3.0{,}5 + 0{,}36.0 = 0{,}32.$$

There is no information about $P(A|B)$ and $P(A|G)$; therefore, both Mistake 1 and Mistake 2 are represented in the solution. To explain $P(A|B) = 0{,}5$, students think that if a twin is picked up at random, the chance of a identical twin is 0,5. We have the same explaination for $P(A|G) = 0{,}5$.

τ12:

A partitioning the sample space Ω into mutually exclusive events such as {identical twin, non-identical twin}.

A: "identical twin".

$\overline{A}$: "non-identical twin".

$\{A, \overline{A}\}$ are mutually exclusive events.

S: "Twin is the same gender".

x is the probability of a identical twin, $1 - x$ is the probability of a non-identical twin.

$$P(S) = P(A)P(S \mid A) + P(\overline{A})P(S \mid \overline{A})$$
$$0{,}64 = x.1 + (1 - x).0{,}5 \Leftrightarrow x = 0{,}28.$$

This is the right answer.

b) Problem 2:

- **Possible strategies**

S2-1: Apply addition probability rule.

S2-2: Apply the total probability rule.

S2-3: Apply the classical probability.

- **Didactic variable**

Variable V: The valuable of component probabilities $P(A_i)$ $(i = 1, ..., n)$ are given or they can be calculated.

In problem 2, there is no information about component probabilities $P(A_i)$ $(i = 1, 2)$ to test the the presence of SL1.

- **Possible sotutions**

S2-1: Apply the additional rule

Technique **τ21**:

A:"Product is defective";

A_1 : "A defective product is in workshop 1";

A_2 : " A defective product is in workshop 2".

Case 1: A defective product is in workshop 1

$$P(A_1) = \frac{100}{1000} = \frac{1}{10}.$$

Case 2: A defective product is in workshop 2

$$P(A_2) = \frac{150}{2000} = \frac{3}{40}.$$

so $P(A) = P(A_1 + A_2) = P(A_1) + P(A_2) = \frac{1}{10} + \frac{3}{40} = \frac{7}{40}.$

This solution shows Mistake 3. The event A:"Product is defective" can not illustrate as $A = A_1 \cup A_2$ because A depends on either workshop 1 or workshop 2. Two events A_1 and A_2 depend on different sample spaces. Moreover, $P(A_1), P(A_2)$ belong to the number of products which are produced by workshops respectively, workshop 1 and workshop 2.

Actually, the requested event A is analyzed to $A = (A \cap A_1) \cup (A \cap A_2)$. Hence, we have a formula $P(A) = P(A \cap A_1) + P(A \cap A_2)$. This formulla is exactly the total probability which is represented in strategy S2-2.

S2-2: Apply the total probability rule

Technique **τ22a**:

A:"Product is defective";

A_1 : "A product is in workshop 1";

A_2 : "A product is in workshop 2".

$\{A_1, A_2\}$ are mutually exclusive events.

$$P(A) = P(A_1)P(A \mid A_1) + P(A_2)P(A \mid A_2)$$
$$= 0,5\frac{100}{1000} + 0,5\frac{150}{2000} = \frac{7}{80}.$$

This solusion above shows Mistake 2. Students think that if a product picked up at random, the product, whose probability is the equal to $\frac{1}{2}$, is in either workshop 1 or workshop 2.

Technique **τ22b**:

A:"Product is defective";

A_1: "A product is in workshop 1";

A_2: " A product is in workshop 2".

$\{A_1, A_2\}$ are mutually exclusive events.

$$P(A) = P(A_1)P(A \mid A_1) + P(A_2)P(A \mid A_2)$$
$$= \frac{1000}{3000}.\frac{100}{1000} + \frac{2000}{3000}.\frac{150}{2000} = \frac{1}{12}.$$

This is a right answer.

S2-3: Apply the classical probability

Technique **τ23**:

A: "Product is defective ".

$$P(A) = \frac{250}{3000} = \frac{1}{12}.$$

This is a right answer.

4.3. A posterior analysis

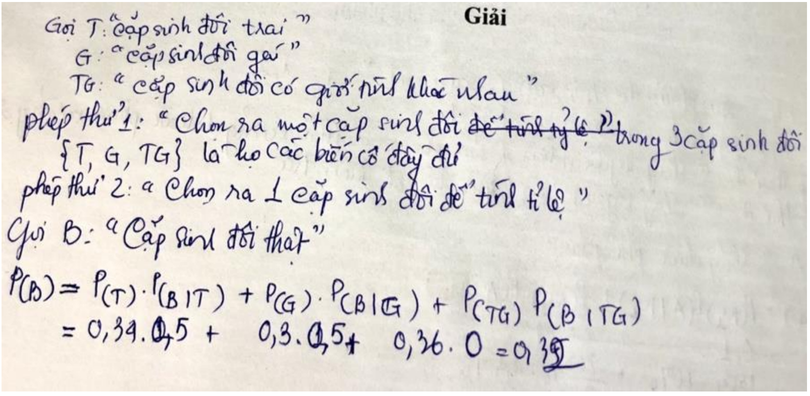
In Hochiminh City, on March 23th 2018, we conducted experiment during 45 minutes on 58 economics and engineering students at Saigon University who are learning Probability Statistics A module. The following are experiment results.

In term of problem 1, 40 in 58 students complete the problem but the remaining 18 students left blank or do not choose the right mutually exclusive events. Among 40 students who solve the whole problem use two techniques which are analyzed above.

Table 3. *Statistics of techniques for problem 1*

Technique τ11	**Technique τ12**
27 (67,5%)	13 (32,5%)

The statistical results show that technique τ11 occupies a dominant percentage with 67,5%. In this technique, students use a partitioning the sample space such as {twins are boys, twins are girls, twins are different gender}. Here are two written papers using technique τ11 of SV16 and SV09 which represent Mistake 1.



Picture 1. *Solution using technique τ11*

Translation of Picture 1: Solution using technique τ11

Solution

T: "Twins are boys".

G: "Twins are girls".

TG: "Twins are different gender".

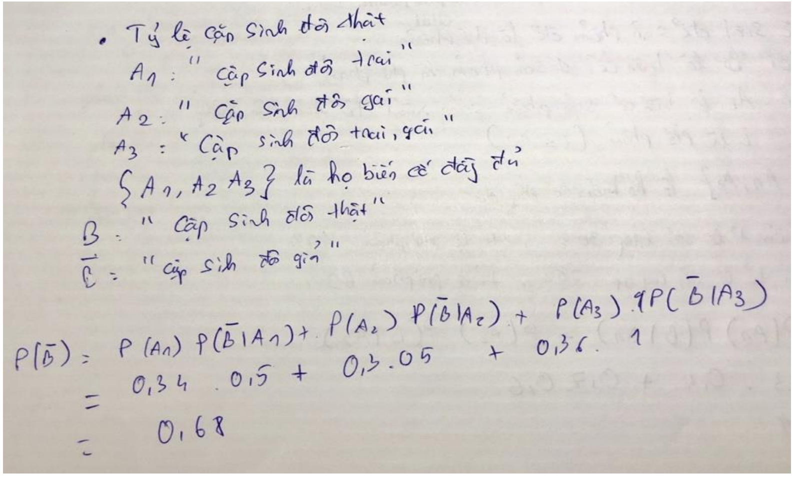
Experiment 1: "Choose one twin among three twins at random"

$\{T, G, TG\}$ are mutually exclusive events.

Experiment 2: "Choose one twin"

B: "Identical twins".

$$P(B) = P(T)P(B|T) + P(G)P(B|G) + P(TG)P(B|TG)$$
$$= 0,34.0,5 + 0,3.0,5 + 0,36.0 = 0,32.$$



Picture 2. *Solution using technique $\tau 11$*

Translation of Picture 2: Solution using technique $\tau 11$

Solution
A_1 : "Twins are boys".
A_2 : "Twins are girls".
A_3 : "Twins are different gender".
$\{A_1, A_2, A_3\}$ are mutually exclusive events.
B : "Identical twins".
$\overline{B}$: "non-identical twins".
$P(\overline{B}) = P(A_1)P(\overline{B}\mid A_1) + P(A_2)P(\overline{B}\mid A_2) + P(A_3)P(\overline{B}\mid A_3)$
$= 0{,}34.0{,}5 + 0{,}3.0{,}5 + 0{,}36.1 = 0{,}68.$

These students choose unsuitable mutually exclusive events in their written papers. Moreover, the papers represent Mistake 2 which was anlalyzed in technique τ11. In term of SV08's paper, student did not compute the requested probability directly but student calculated the probability through non-identical twins' rate. However, although student had the non-identical twins' rate, student did not do the final subtraction to have the final result as SV16.

Among 13 students use technique τ12. There are only 8 students solve the whole problem and have the right answers.

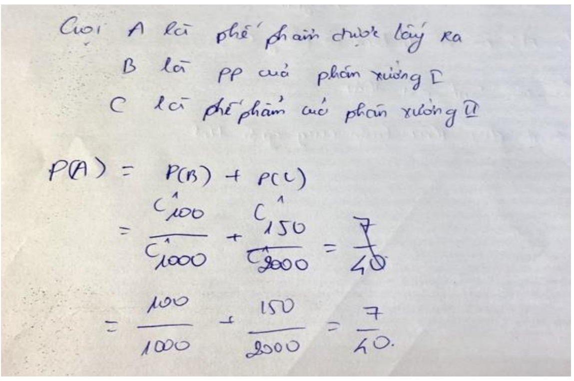
The result shows that the Mistake 1 presence is in choosing a unsuitable partitioning of a sample space.

For the problem 2, 43 in 58 students solve the problem and the remaining 18 students left blank or do not complete.

Table 4. *Statistics of strategies for problem 2*

Strategy S2-1	**Strategy S2-2**		**Strategy S2-3**	**Others Strategy**
τ21	**τ22a**	**τ22b**	**τ23**	
7 (16,28%)	10 (23,25%)	25 (58,14%)	1 (2,33%)	0 (0%)

Strategy S2-2 is dominant in the problem 2. However, both technique τ22b and τ23 give the right answer. In this experiment, there are 17 students gave the wrong answer. Among them, there are 7 students used strategy S2-1 with the technique τ21 show Mistake 3; there are 10 students used strategy S2-2 with the technique τ22a show Mistake 2. Here are two written papers of SV55 and SV38 represent two mistakes respectively Mistake 3 and Mistake 2.



Picture 3. *Solution using technique τ21*

Translation of Picture 3: Solution using technique τ21

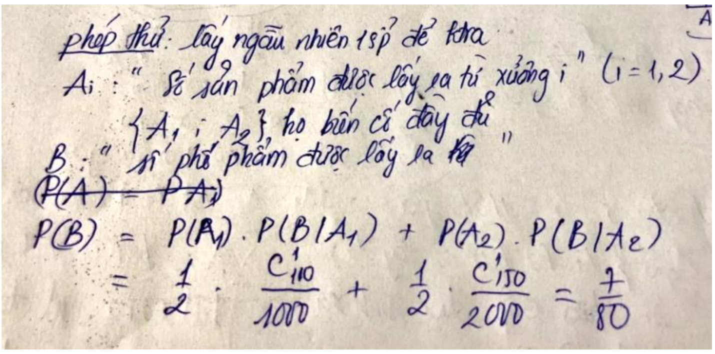
Solution

A:"Product is defective";

B:"A defective product is in workshop 1";

C:" A defective product is in workshop 2".

$$P(A) = P(B) + P(C) = \frac{C_{100}^1}{C_{1000}^1} + \frac{C_{150}^1}{C_{2000}^1} = \frac{100}{1000} + \frac{150}{2000} = \frac{7}{40}.$$



Picture 4. *Solution using technique* $\tau 22a$

Translation of Picture 4: Solution using technique $\tau 22a$

Solution
Experiment: "Choose one product to check at random".
A_i : "A product is in workshop i" $(i = 1, 2)$;
A : "Product is defective";
$\{A_1, A_2\}$ are mutually exclusive events.
$P(B) = P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)$
$= \frac{1}{2}\frac{C_{100}^1}{1000} + \frac{1}{2}\frac{C_{150}^1}{2000} = \frac{7}{80}.$

These assignments made mistakes that we have analyzed in technique τ21 and τ22a.

According to the experiment results of both problem 1 and 2, we confirm that there are Mistake 1, Mistake 2 and Mistake 3 in learning the total probability of economics and engineering students at Saigon university. These are the mistakes that we have anticipated through a analysis from the syllabus institution for economics and engineering students. It shows that the hypothesis H is correct.

5. Conclusion

The results show that economics and engineering students at Saigon unviversity make Mistake 1, Mistake 2 and Mistake 3 when they do the T-task related to the total probability. Especially, Mistake 1 and Mistake 3 come from didactic obstacle. Mistake 1 relates to didactic contract **R** which contains an implicit convention in choosing mutually exclusive subsets. They either are given the percentage (%) or are calculated by some simple steps. Mistake 3 comes from the situations in the syllabus where all of exercises and examples in both TB and WB illustrate that all of mutually exclusive events are generated at the same experiment. Therefore, they have the same sample space.

The results in the article aslo raise a problem related to obstacles and mistakes of economics and engineering students in learning the total probability for teacher training quality and teachers'universities.

❖ ***Conflict of Interest:*** *Authors have no conflict of interest to declare.*

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