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Research Article

THE BOUNDEDNESS OF GENERALIZED WEIGHTED HARDY-CESÀRO OPERATORS ON GENERALIZED MORREY SPACES

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ABSTRACT

Let $\psi:[0,1]\to[0,\infty)$, $s:[0,1]\to\mathbb{R}$ *be measurable functions and* Γ *be a parameter curve in* $\mathbb{R}^n$ *given by* $(t,x)\in[0,1]\times\mathbb{R}^n\mapsto s(t,x)=s(t)x$. *In this paper, we study the boundedness of the weighted Hardy-Cesàro operator defined by* $U_{\psi,s}f(x)=\int_0^1 f\left(s(t)x\right)\psi(t)dt$, *for measurable complex-valued functions* f *on* $\mathbb{R}^n$, *on generalized Morrey spaces* $M_{p,\varphi}$. *We obtain some sufficient conditions on the functions* s, ψ *and* φ, *which ensure the boundedness of the weighted Hardy-Cesàro operator and its commutator with symbols in BMO spaces on generalized Morrey spaces* $M_{p,\varphi}$.

Keywords: weighted Hardy-Cesàro operator; commutator; generalized Morrey space; BMO space

1. Introduction

Consider the classical Hardy operator U defined by

$$Uf(x)=\frac{1}{x}\int_0^x f(t)dt, x\neq 0$$

for $f\in L^1_{\mathrm{loc}}(\mathbb{R})$. A celebrated Hardy integral inequality, see (Hardy, Littlewood, & Polya, 1952), can be formulated as

$$\|Uf\|_{L^p(\mathbb{R})}\leq\frac{p}{p-1}\|f\|_{L^p(\mathbb{R})},$$

where $1<p<\infty$, in which the constant $\dfrac{p}{p-1}$ is known as the best constant.

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The Hardy integral inequality and its variants have played an important role in various branches of analysis such as approximation theory, differential equations, and the theory of function spaces. Therefore, the Hardy integral inequalities for operator U and their generalizations have been studied extensively.

The generalized Hardy operator was first introduced by C. Carton-Lebrun and M. Fosset in (Carton-Lebrun, & Fosset, 1984), in which the authors defined the weighted Hardy operator U_ψ as follows. Let $\psi:[0,1]\to[0,\infty)$ be a measurable function, and let f be a measurable complex-valued function on $\mathbb{R}^n$. Then the weighted Hardy operator U_ψ is defined by

$$U_\psi f(x) = \int_0^1 f(tx)\psi(t)dt,\ x \in \mathbb{R}^n.$$

Under certain conditions on ψ, C. Carton-Lebrun and M. Fosset (1984) showed that U_ψ is bounded from $BMO(\mathbb{R}^n)$ into itself. Moreover, U_ψ commutes with the Hilbert transform in the case $n=1$ and with certain Calderón-Zygmund singular integral operators (and thus with the Riesz transforms) in the case $n \geq 2$.

Later, in (Xiao, 2001), J. Xiao obtained that U_ψ is bounded on $L^p(\mathbb{R}^n)$ if and only if

$$\mathcal{A} := \int_0^1 t^{-n/p}\psi(t)dt < \infty$$

and showed the interesting estimate that the corresponding operator norm is exactly $\mathcal{A}$. J. Xiao (2001) also obtained the $BMO(\mathbb{R}^n)-$bounds of U_ψ, which sharpened the main result in (Carton-Lebrun, & Fosset, 1984).

Recently, Z. W. Fu, Z. G. Liu, and S. Z. Lu (2009) gave a necessary and sufficient condition on the weight function ψ, which ensures the boundedness of the commutators of weighted Hardy operators U_ψ on $L^p(\mathbb{R}^n)$, $1<p<\infty$, with symbols in $BMO(\mathbb{R}^n)$ in (Fu, Liu, & Lu, 2009). Since then, some authors have investigated bounds of U_ψ and its commutator on classical Morrey spaces, Campanato spaces, Triebel-Lizorkin-type spaces (see (Fu, & Lu, 2010), (Kuang, 2010), and (Tang, & Zhou, 2012)).

Motivated by all of the above-mentioned facts, we consider the generalized weighted *Hardy-Cesàro* operator and its commutator as follows.

Definition 1.1.

Let $\psi:[0,1]\to[0,\infty)$ and $s:[0,1]\to\mathbb{R}$ be measurable functions. Then the generalized weighted *Hardy-Cesàro* operator $U_{\psi,s}$, associated to the parameter curve $s(x,t):=s(t)x$, is defined by

$$U_{\psi,s}f(x) = \int_0^1 f\left(s(t)x\right)\psi(t)dt,$$

for measurable complex-valued functions f on $\mathbb{R}^n$.

Definition 1.2. Let b be a locally integrable function on $\mathbb{R}^n$. The commutators of b and the operator $U_{\psi,s}$ is defined by

$$U^b_{\psi,s} f = bU_{\psi,s}(f) - U_{\psi,s}(bf).$$

Our aim in this paper is to study norm inequalities for the generalized weighted *Hardy-Cesàro* operator $U_{\psi,s}$ and its commutator $U^b_{\psi,s}$ with symbols b being BMO functions on generalized Morrey spaces $M_{p,\varphi}$, which are introduced by T. Mizuhara in (Mizuhara, 1991) as follows.

Definition 1.3.

Let φ be a positive measurable function on $\mathbb{R}^n \times (0,\infty)$ and $0 < p < \infty$. Then the generalized Morrey space $M_{p,\varphi} = M_{p,\varphi}(\mathbb{R}^n)$ is defined by

$$M_{p,\varphi} = \left\{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{M_{p,\varphi}} = \sup_{x\in\mathbb{R}^n, r>0} [\varphi(x,r)]^{-1} \|f\|_{L^p(B(x,r))} < \infty \right\},$$

and the generalized central Morrey space $M^{cen}_{p,\varphi} = M^{cen}_{p,\varphi}(\mathbb{R}^n)$ is defined by

$$M^{cen}_{p,\varphi} = \left\{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{M^{cen}_{p,\varphi}} = \sup_{r>0} [\varphi(0,r)]^{-1} \|f\|_{L^p(B(0,r))} < \infty \right\}.$$

Obviously, the above definition recovers the definition of classical Morrey spaces $L^{p,\lambda}(\mathbb{R}^n)$ if we choose $\varphi(x,r) = r^{\frac{\lambda}{p}}$.

Specifically, we present some sufficient conditions imposed on the functions s, ψ and φ in order to obtain the boundedness of the weighted Hardy-Cesàro operator $U_{\psi,s}$ and its commutator on generalized Morrey spaces $M_{p,\varphi}$. These results extend the results in (Xiao, 2001), (Fu, Liu, & Lu, 2009) and (Fu, & Lu, 2010) in some sense.

Throughout the paper, the letter C is used to denote (possibly different) constants that are independent of the essential variables. We also denote a ball centered at x of radius r and its Lebesgue measure by $B(x,r)$ and $|B(x,r)|$, respectively.

2. Main results

Throughout this section, we assume that the function φ satisfies the following condition.

Definition 2.1.

Let $\alpha > 0$ and φ be a positive measurable function on $\mathbb{R}^n \times (0,\infty)$. We say $\varphi \in SH^{\alpha}(\mathbb{R}^n \times (0,\infty))$ if there exists a positive constant C such that for all $(x,r) \in \mathbb{R}^n \times (0,\infty)$ and for all $t \in (0,\infty)$, one has $\varphi(tx,tr) \le Ct^{\alpha}\varphi(x,r)$.

Some examples of such functions are $\varphi(x,r) = r^{\alpha}$ or are homogeneous functions of degree α.

Our first main result in this section is formulated as follows.

Theorem 2.2.

Let $0 < p < \infty$, *let* $s, \psi : [0,1] \to [0,\infty)$ *be measurable functions such that* $s(t) > 0$ *a.e.* $t \in [0,1]$ *and let* $\varphi \in SH^{\alpha}(\mathbb{R}^n \times (0,\infty))$. *Then* $U_{\psi,s}$ *is bounded on* $M_{p,\varphi} = M_{p,\varphi}(\mathbb{R}^n)$, *provided that* $\int_0^1 s(t)^{\alpha-\frac{n}{p}}\psi(t)dt < \infty.$

Proof. Suppose that $\int_0^1 s(t)^{\alpha-\frac{n}{p}}\psi(t)dt < \infty.$

For any $f \in M_{p,\varphi}$, $x \in \mathbb{R}^n$ and $r > 0$, it follows from Minkowski inequality that

$$[\varphi(x,r)]^{-1}\left(\int_{B(x,r)} \left|U_{\psi,s}f(y)\right|^p dy\right)^{\frac{1}{p}} = [\varphi(x,r)]^{-1}\left(\int_{B(x,r)} \left|\int_0^1 f\left(s(t)y\right)\psi(t)dt\right|^p dy\right)^{\frac{1}{p}}$$

$$\le [\varphi(x,r)]^{-1}\int_0^1\left(\int_{B(x,r)} \left|f\left(s(t)y\right)\right|^p dy\right)^{\frac{1}{p}}\psi(t)dt$$

$$= [\varphi(x,r)]^{-1}\int_0^1\left(\int_{B(s(t)x,s(t)r)} \left|f\left(y\right)\right|^p dy\right)^{\frac{1}{p}} s(t)^{-\frac{n}{p}}\psi(t)dt$$

$$\le \int_0^1 \frac{\varphi(s(t)x,s(t)r)}{\varphi(x,r)}[\varphi(s(t)x,s(t)r)]^{-1}\left(\int_{B(s(t)x,s(t)r)} \left|f\left(y\right)\right|^p dy\right)^{\frac{1}{p}} s(t)^{-\frac{n}{p}}\psi(t)dt$$

$$\le \|f\|_{M_{p,\varphi}}\int_0^1 \frac{\varphi(s(t)x,s(t)r)}{\varphi(x,r)} s(t)^{-\frac{n}{p}}\psi(t)dt \le C\|f\|_{M_{p,\varphi}}\int_0^1 s(t)^{\alpha-\frac{n}{p}}\psi(t)dt,$$

where the last inequality comes from the assumption that $\varphi \in SH^{\alpha}(\mathbb{R}^n \times (0,\infty))$.

Clearly, the estimates above imply

$$\left\|U_{\psi,s}f\right\|_{M_{p,\varphi}} \le C\left\|f\right\|_{M_{p,\varphi}} \int_0^1 s(t)^{\alpha-\frac{n}{p}}\psi(t)dt.$$

In other words, $U_{\psi,s}$ is defined as a bounded operator on $M_{p,\varphi}$ and

$$\left\|U_{\psi,s}\right\|_{M_{p,\varphi}\to M_{p,\varphi}} \le C\int_0^1 s(t)^{\alpha-\frac{n}{p}}\psi(t)dt,$$

which completes the proof of Theorem 2.2.

Analogous to the proof of Theorem 2.2, we can present a sufficient condition such that the integral operator $\mathcal{U}_{\psi,s}$, which is defined by

$$\mathcal{U}_{\psi,s}f(x) = \int_0^\infty f\left(s(t)x\right)\psi(t)dt,$$

is bounded on $M_{p,\varphi}$.

Theorem 2.3.

Let $0 < p < \infty$, *let* s, $\psi : [0,\infty) \to [0,\infty)$ *be measurable functions such that* $s(t) > 0$ *a.e.* $t \in [0,\infty)$ *and let* $\varphi \in SH^\alpha(\mathbb{R}^n \times (0,\infty))$. *Then* $\mathcal{U}_{\psi,s}$ *is bounded on* $M_{p,\varphi} = M_{p,\varphi}(\mathbb{R}^n)$, *provided that* $\int_0^\infty s(t)^{\alpha-\frac{n}{p}}\psi(t)dt < \infty.$

Before coming to the second main result in this section, let us recall here the definition of spaces $BMO(\mathbb{R}^n)$ which are first introduced by F. John and L. Nirenberg in (John, & Nirenberg, 1961).

Definition 2.4.

Given a function $f \in L^1_{\text{loc}}(\mathbb{R}^n)$ and a ball B in $\mathbb{R}^n$, let f_B denote the average of f on B, that is

$$f_B = \frac{1}{|B|}\int_B f(y)dy.$$

Define the sharp maximal function by

$$M^\sharp f(x) = \sup_B \frac{1}{|B|}\int_B |f - f_B|,$$

where the supremum is taken over all balls B containing x.

We say that f has *bounded mean oscillation* if the function $M^\sharp$ is bounded. The space of functions with this property is denoted by $BMO(\mathbb{R}^n)$, that is

$$BMO(\mathbb{R}^n) = \left\{f \in L^1_{\text{loc}}(\mathbb{R}^n) : M^\sharp f \in L^\infty\right\}.$$

We define a norm on $BMO(\mathbb{R}^n)$ by $\|f\|_{BMO} = \|M^\sharp f\|_\infty$.

In the sequent, we will need the following two important results relating to BMO spaces.

Proposition 2.5.

For all $1 < p < \infty$, *we have*

$$\|f\|_{BMO,p} = \sup_B \left(\frac{1}{|B|} \int_B |f(x) - f_B|^p \, dx \right)^{1/p}$$

is a norm equivalent to the BMO norm.

Proof. We refer the reader to the proof of Proposition 2.5 in (Duoandikoetxea, 2000).

Proposition 2.6.

There exists a positive constant C *such that for any* $B_1 = B(x_1, r_1)$ *and* $B_2 = B(x_2, r_2)$ *in* $\mathbb{R}^n$, *whose intersection is not empty and* $\frac{1}{2} r_2 \le r_1 \le 2r_2$, *we have* $|B| \le C|B_1|$ *and* $|B| \le C|B_2|$, *where* B *is the smallest ball containing both* B_1 *and* B_2. *Moreover, for any function* $b \in BMO(\mathbb{R}^n)$, *then the following inequality holds*

$$|b_{B_1} - b_{B_2}| \le 2C \| b \|_{BMO}.$$

Proof. Obviously, there exists a positive constant C_1 such that $|B(x, 2r)| = C_1 |B(x, r)|$, for any $x \in \mathbb{R}^n$ and $r > 0$. Let $B_1 = B(x_1, r_1)$ and $B_2 = B(x_2, r_2)$, whose intersection is not empty and $\frac{r_2}{2} \le r_1 \le 2r_2$. Without loss of generality, one assumes that $r_2 \le r_1 \le 2r_2$.

Take $x \in B_1 \cap B_2$. Then, we have

$$|B| \le |B(x, 2r_1)| \le C_1 |B(x, r_1)| \le C_1 |B(x_1, 2r_1)| \le C_1^2 |B_1|,$$

and

$$|B| \le |B(x, 4r_2)| \le C_1^2 |B(x, r_2)| \le C_1^3 |B(x_2, r_2).|$$

Hence we can choose the constant $C = \max\{C_1^2, C_1^3\}$.

Moreover, for any function $b \in BMO(\mathbb{R}^n)$, note that

$$|b_{B_1} - b_{B_2}| \le |b_{B_1} - b_B| + |b_{B_2} - b_B|.$$

It is clear to see that

$$\left|b_B - b_{B_1}\right| = \left|b_B - \frac{1}{|B_1|}\int_{B_1} b(y)dy\right| \le \frac{1}{|B_1|}\int_{B_1}|b(y)-b_B|dy \le \frac{C}{|B|}\int_B |b(y)-b_B|dy \le C\,\|b\|_{BMO}.$$

The left term is estimated in a similar way. Eventually, we complete the proof.

We are now ready to state the following main result.

Theorem 2.7.

Let $1<p<q<\infty$, *let* $s,\psi:[0,1]\to[0,\infty)$ *be measurable functions such that* $0<s(t)\le 1$ *a.e.* $t\in[0,1]$, *let* $\varphi\in SH^{\alpha}(\mathbb{R}^n\times(0,\infty))$ *and* $b\in BMO(\mathbb{R}^n)$. *Then* $U^b_{\psi,s}$ *is bounded from* $M^{cen}_{p,\varphi}\cap L^q_{loc}(\mathbb{R}^n)$ *to* $M^{cen}_{p,\varphi}$, *provided that* $\int_0^1 s(t)^{\alpha-\frac{n}{p}}\left(2-\log_2 s(t)\right)\psi(t)dt<\infty.$

Proof. Assume that $\int_0^1 s(t)^{\alpha-\frac{n}{q}}\left(2-\log_2 s(t)\right)\psi(t)dt<\infty.$

Let B be any ball centered at the origin of radius r, and let f be any function in $M^{cen}_{p,\varphi}\cap L^q_{loc}$. Then it follows from the Minkowski inequality that

$$I=\left(\varphi(0,r)^{-p}\int_B\left|U^b_{\psi,s}f(y)\right|^p dy\right)^{\frac{1}{p}} \le \int_0^1\left(\varphi(0,r)^{-p}\int_B\left|(b(y)-b(s(t)y))f(s(t)y)\right|^p dy\right)^{\frac{1}{p}}\psi(t)dt.$$

Applying the following elementary inequality

$$3^{p-1}\left(|x|^p+|y|^p+|z|^p\right)\ge|x+y+z|^p,\; x,\,y,\,z\in\mathbb{C}$$

to the right-hand side of the above inequality gives

$$I\le C(I_1+I_2+I_3),$$

where

$$I_1=\int_0^1\left(\varphi(0,r)^{-p}\int_B\left|(b(y)-b_B)f(s(t)y)\right|^p dy\right)^{\frac{1}{p}}\psi(t)dt,$$

$$I_2=\int_0^1\left(\varphi(0,r)^{-p}\int_B\left|(b_{s(t)B}-b_B)f(s(t)y)\right|^p dy\right)^{\frac{1}{p}}\psi(t)dt,$$

$$I_3=\int_0^1\left(\varphi(0,r)^{-p}\int_B\left|(b(s(t)y)-b_{s(t)B})f(s(t)y)\right|^p dy\right)^{\frac{1}{p}}\psi(t)dt,$$

and the constant C depends only on p.

Let us now estimate the term I_1. For any $0<\varepsilon<q-p$, set $l=p+\varepsilon$ and $l^*=\dfrac{lp}{l-p}$. Then applying the Holder inequality with the pair $\left(k=\dfrac{l}{l-p}, k'=\dfrac{l}{p}\right)$ for the term I_1 yields

$$I_1\le C\varphi(0,r)^{-1}|B|^{\frac{1}{p}}\int_0^1\left(\frac{1}{|B|}\int_B|f(s(t)y)|^{p+\varepsilon}\,dy\right)^{\frac{1}{p+\varepsilon}}\left(\frac{1}{|B|}\int_B|b(y)-b_B|^{l^*}\,dy\right)^{\frac{1}{l^*}}\psi(t)dt.$$

In view of Proposition 2.5, we deduce that

$$I_1\le C\|b\|_{BMO}|B|^{\frac{1}{p}-\frac{1}{p+\varepsilon}}\varphi(0,r)^{-1}\int_0^1\left(\int_B|f(s(t)y)|^{p+\varepsilon}\,dy\right)^{\frac{1}{p+\varepsilon}}\psi(t)dt\,.$$

Note that $f\in L^p_{loc}(\mathbb{R}^n)\cap L^q_{loc}(\mathbb{R}^n)$, so letting $\varepsilon\to 0^+$ from the preceding estimate yields

$$\begin{aligned}I_1&\le C\|b\|_{BMO}\,\varphi(0,r)^{-1}\int_0^1\left(\int_B|f(s(t)y)|^{p}\,dy\right)^{\frac{1}{p}}\psi(t)dt\\&\le C\|b\|_{BMO}\|f\|_{M^{cen}_{p,\varphi}}\int_0^1 s(t)^{\alpha-\frac{n}{p}}\psi(t)dt\\&\le C\|b\|_{BMO}\|f\|_{M^{cen}_{p,\varphi}}\int_0^1 s(t)^{\alpha-\frac{n}{p}}\left(2-\log_2 s(t)\right)\psi(t)dt.\end{aligned}$$

Similarly, one can use the same argument above to obtain $I_3\le C\|b\|_{BMO}\|f\|_{M^{cen}_{p,\varphi}}\int_0^1 s(t)^{\alpha-\frac{n}{p}}\left(2-\log_2 s(t)\right)\psi(t)dt.$

For the last term I_2, let us express this term as

$$\begin{aligned}I_2&=\int_0^1\left(\varphi(0,r)^{-p}\int_B|f(s(t)y)|^p\,dy\right)^{\frac{1}{p}}\left|b_B-b_{s(t)B}\right|\psi(t)dt\\&\le C\|f\|_{M^{cen}_{p,\varphi}}\int_0^1\left|b_B-b_{s(t)B}\right|s(t)^{\alpha-\frac{n}{p}}\psi(t)dt\\&\le C\|f\|_{M^{cen}_{p,\varphi}}\sum_{m=0}^{\infty}\int_{\left\{t\in[0,1]:2^{-m-1}\le s(t)\le 2^{-m}\right\}}\left|b_B-b_{s(t)B}\right|s(t)^{\alpha-\frac{n}{p}}\psi(t)dt.\end{aligned}$$

At this stage, observe that for each $m \in \mathbb{N}$, we have

$$\left|b_B - b_{s(t)B}\right| \leq \sum_{i=0}^{m} \left|b_{2^{-i-1}B} - b_{2^{-i}B}\right| + \left|b_{2^{-m-1}B} - b_{s(t)B}\right|.$$

Therefore, in the light of Proposition 2.6, we deduce that

$$I_2 \leq C\|b\|_{BMO}\|f\|_{M^{cen}_{p,\varphi}} \sum_{m=0}^{\infty} \int\limits_{\{t\in[0,1]:2^{-m-1}\leq s(t)\leq 2^{-m}\}} (m+2)s(t)^{\alpha-\frac{n}{p}}\psi(t)dt$$

$$\leq C\|b\|_{BMO}\|f\|_{M^{cen}_{p,\varphi}} \sum_{m=0}^{\infty} \int\limits_{\{t\in[0,1]:2^{-m-1}\leq s(t)\leq 2^{-m}\}} \left(2-\log_2 s(t)\right)s(t)^{\alpha-\frac{n}{p}}\psi(t)dt$$

$$\leq C\|b\|_{BMO}\|f\|_{M^{cen}_{p,\varphi}} \int_0^1 \left(2-\log_2 s(t)\right)s(t)^{\alpha-\frac{n}{p}}\psi(t)dt,$$

which, combined with the last estimates of I_1 and I_3 above completes the proof of Theorem 2.7.

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TÍNH BỊ CHẶN CỦA TOÁN TỬ HARDY-CESÀRO CÓ TRỌNG TỔNG QUÁT TRÊN CÁC KHÔNG GIAN MORREY TỔNG QUÁT

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TÓM TẮT

Giả sử $\psi:[0,1]\to[0,\infty)$, $s:[0,1]\to\mathbb{R}$ *là các hàm đo được và* Γ *là một đường cong tham số trong* $\mathbb{R}^n$ *được xác định bởi* $(t,x)\in[0,1]\times\mathbb{R}^n \mapsto s(t,x)=s(t)x.$ *Trong bài báo này, chúng tôi nghiên cứu tính bị chặn của toán tử Hardy-Cesàro có trọng* $U_{\psi,s}f(x)=\int_0^1 f\left(s(t)x\right)\psi(t)dt,$ *trong đó* f *là một hàm đo được trên* $\mathbb{R}^n$, *trên các không gian Morrey tổng quát* $M_{p,\varphi}$. *Chúng tôi thiết lập được một số điều kiện đủ trên các hàm* s ψ *và* φ*, mà các điều kiện này đảm bảo tính bị chặn của toán tử Hardy-Cesàro và hoán tử của nó trên các không gian Morrey tổng quát* $M_{p,\varphi}$ *khi các biểu tượng thuộc không gian BMO.*

Từ khóa: toán tử Hardy-Cesàro có trọng; hoán tử; không gian Morrey tổng quát; không gian BMO