# SEMICONTINUITY OF SOLUTIONS OF PARAMETRIC SCALAR QUASIVARIATIONAL INEQUALITY PROBLEMS OF THE MINTY TYPE

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**ABSTRACT**

*In this paper, we study two kinds of parametric scalar quasivariational inequality problems of the Minty type (in short, ($MQIP_1$) and ($MQIP_2$)). After then, we discuss the the upper semicontinuity, the lower semicontinuity, the Hausdorff lower semicontinuity the continuity and H-continuity for these problems. The results presented in this paper are improve and extend some main results of Lalitha and Bhatia [J. Optim. Theory. Appl. 148, 281--300 (2011)]. Some examples are given to illustrate our results.*

***Keywords:*** Parametric quasivariational inequality problems of the Minty type. Upper semicontinuity, Compactness, Closedness, Lower semicontinuity, Hausdorff lower semicontinuity, Continuity, Hausdorff continuity.

**TÓM TẮT**

***Tính chất nửa liên tục của các nghiệm của các bài toán tựa bất đẳng thức biến phân vô hướng phụ thuộc tham số loại Minty***

*Trong bài báo này, chúng tôi nghiên cứu hai loại bài toán bất đẳng thức tựa biến phân phụ thuộc tham số loại Minty (viết tắt, ($MQIP_1$) và ($MQIP_2$)). Sau đó, chúng tôi thảo luận tính nửa liên tục trên, nửa liên tục dưới, nửa liên tục dưới Hausdorff, liên tục và tính liên tục Hausdorff cho các bài toán này. Kết quả hiện tại trong bài báo là cải thiện và mở rộng một số kết quả chính của Lalitha và Bhatia [J. Optim. Theory. Appl. 148, 281--300 (2011)]. Một số ví dụ được đưa ra để minh chứng cho các kết quả của chúng tôi.*

***Từ khóa:*** các bài toán tựa bất đẳng thức biến phân loại Minty phụ thuộc tham số, tính nửa liên tục trên, tính nửa liên tục dưới, tính nửa liên tục dưới Hausdorff, tính liên tục, liên tục Hausdorff.

## 1. Introduction and Preliminaries

A vector variational inequality problem was first introduced and studied by Giannessi [4] in the setting of finite-dimensional Euclidean spaces. Since then, many authors have investigated vector variational inequality problems in abstract spaces, see [1, 3] and the references therein. Recently, Lalitha and Bhatia [6] have considered a parametric scalar quasivariational inequality problem of the Minty type, and kinds of the semicontinuity are also obtained. Motivated by research works mentioned above, in this paper, we introduce two kinds of parametric quasivariational inequality problems of the Minty type in Hausdorff topological vector spaces.

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Let X, Y be two Hausdorff topological vector spaces and $\Gamma, \Lambda$ be two topological vector spaces. Let $L(X,Y)$ be the space of all linear continuous operators from $X$ into $Y$, and $A \subset X$ be a nonempty subset. Let $K_1 : A\times\Gamma \to 2^A, K_2 : A\times\Gamma \to 2^A$ and $T : A\times\Gamma \to 2^{L(X,Y)}$ are set-valued mappings. And let $\psi : A\times A\times\Lambda \to A$ be continuous single-valued mapping. Denoted $\langle z, x\rangle$ by the value of a linear operator $z \in L(X;Y)$ at $x \in X$, we always assume that $\langle .,.\rangle$ is continuous.

We consider the following parametric quasivariational inequality problems of the Minty type (in short, (MQIP$_1$) and (MQIP$_2$)), respectively.

**(MQIP$_1$)** Find $\bar{x} \in K_1(\bar{x},\gamma)$ such that $\exists z \in T(y,\gamma)$ and

$\langle z,\psi(y,\bar{x},\lambda)\rangle \geq 0, \forall y \in K_2(\bar{x},\gamma).$

**(MQIP$_2$)** Find $\bar{x} \in K_1(\bar{x},\gamma)$ such that $\forall z \in T(y,\gamma)$ and

$\langle z,\psi(y,\bar{x},\lambda)\rangle \geq 0, \forall y \in K_2(\bar{x},\gamma).$

For each $\gamma \in \Gamma, \lambda \in \Lambda$, and let $E(\gamma) := \{x \in A : x \in K_1(x,\gamma)\}$. We denote $S_1(\gamma,\lambda)$ and $S_2(\gamma,\lambda)$ are solution sets of (MQIP$_1$) and (MQIP$_2$), respectively. By the definition, the following relation is clear: $S_2(\gamma,\lambda) \subseteq S_1(\gamma,\lambda)$.

Throughout the article, we assume that $S_1(\gamma,\lambda) \neq \emptyset$ and $S_2(\gamma,\lambda) \neq \emptyset$ for each $(\gamma,\lambda)$ in the neighborhoods $(\gamma_0,\lambda_0) \in \Gamma\times\Lambda$.

Now we recall some notions (see, [1-5]). Let $X$ and $Z$ be as above and $G : X \to 2^Z$ be a multifunction. $G$ is said to be lower semicontinuous (lsc) at $x_0$ if $G(x_0) \cap U \neq \emptyset$ for some open set $U \subseteq Z$ implies the existence of a neighborhood $N$ of $x_0$ such that, for all $x \in N, G(x) \cap U \neq \emptyset$. An equivalent formulation is that: $G$ is lsc at $x_0$ if $\forall x_\alpha \to x_0$, $\forall z_0 \in G(x_0), \exists z_\alpha \in G(x_\alpha), z_\alpha \to z_0$. $G$ is called upper semicontinuous (usc) at $x_0$ if for each open set $U \supseteq G(x_0)$, there is a neighborhood $N$ of $x_0$ such that $U \supseteq G(N)$. $G$ is said to be Hausdorff upper semicontinuous (H-usc in short; Hausdorff lower semicontinuous, H-lsc, respectively) at $x_0$ if for each neighborhood $B$ of the origin in $Z$, there exists a neighborhood $N$ of $x_0$ such that, $G(x) \subseteq G(x_0) + B, \forall x \in N$ ($G(x_0) \subseteq G(x) + B, \forall x \in N$). $G$ is said to be continuous at $x_0$ if it is both lsc and usc at $x_0$ and to be H-continuous at $x_0$ if it is both H-lsc and H-usc at $x_0$. $G$ is called closed at $x_0$ if for each net $\{(x_\alpha, z_\alpha)\} \subseteq \text{graph}G := \{(x,z) \mid z \in G(x)\}, (x_\alpha, z_\alpha) \to (x_0, z_0)$, $z_0$ must belong to $G(x_0)$. The closedness is closely related to the upper (and Hausdorff upper) semicontinuity.

***Lenmma 1.1.*** *([2])*

*Let $X$ and $Z$ be two topological vector spaces and $G: X \to 2^Z$ be a multifunction.*

*(i) If $Z$ is compact and $G$ is closed at $x_0$, then $G$ is usc at $x_0$;*

*(ii) If $G$ is usc at $x_0$ and $G(x_0)$ is closed, then $G$ is closed at $x_0$;*

*(iii) If $G$ is usc at $x_0$ then $G$ is $H$-usc at $x_0$. Conversely if $G$ is $H$-usc at $x_0$ and if $G(x_0)$ compact, then $G$ usc at $x_0$;*

*iv) If $G$ is H-lsc at $x_0$ then $G$ is lsc. The converse is true if $G(x_0)$ is compact;*

## 2. Upper semicontinuity of solution sets

In this section, we discuss the upper semicontinuity of the solution sets for the problems (MQIP$_1$) and (MQIP$_2$).

***Theorem 2.1.***

*Assume for the problem $(MQIP_1)$ that*

*(i) $E$ is usc at $\gamma_0$ and $E(\gamma_0)$ compact set;*

*(ii) in $K_1(A,\Gamma)\times\{\gamma_0\}$, $K_2$ is lsc;*

*(iii) in $K_2(K_1(A,\Gamma),\Gamma)\times\{\gamma_0\}$, $T$ is usc and compact-valued.*

*Then, $S_1$ is usc at $(\gamma_0,\lambda_0)$. Moreover, $S_1(\gamma_0,\lambda_0)$ is compact and $S_1$ is closed at $(\gamma_0,\lambda_0)$.*

***Proof.***

We first prove that $S_1$ is upper semicontinuous at $(\gamma_0,\lambda_0)$. Indeed, we suppose to the contrary the existence of an open subset $U$ of $S_1(\gamma_0,\lambda_0)$ such that for all $\{(\gamma_n,\lambda_n)\}$ convergent to $\{(\gamma_0,\lambda_0)\}$, there is $x_n \in S_1(\gamma_n,\lambda_n)$, $x_n \notin U$, for al $n$. Since $E$ is usc and compact-valued at $\gamma_0$, we can assume that $x_n$ tends to $x_0$ for some $x_0 \in E(\gamma_0)$. If $x_0 \notin S_1(\gamma_0,\lambda_0)$, $\exists y_0 \in K_2(x_0,\gamma_0), \forall z_0 \in T(y_0,\gamma_0)$ such that

$$\langle z_0, \psi(y_0,x_0,\lambda_0)\rangle \ngeq 0.$$

By the lower semicontinuity of $K_2$ at $(x_0,\gamma_0)$, there exists $y_n \in K_2(x_n,\gamma_n)$ such that $y_n \to y_0$. Since $x_n \in S_2(\gamma_n,\lambda_n)$, $\exists z_n \in T(y_n,\gamma_n)$ such that

$$\langle z_n, \psi(y_n,x_n,\lambda_n)\rangle \geq 0. \quad (2.1)$$

Since $T$ is usc and compact-valued at $(y_0,\gamma_0)$, there exists $z_0 \in T(y_0,\gamma_0)$ such that $z_n \to z_0$ (can take a subnet if necessary). On the other hand, by the continuity of $\psi$ and $\langle .,. \rangle$, hence it follows from (2.1) that

$$\langle z_0, \psi(y_0,x_0,\lambda_0)\rangle \geq 0,$$

it is impossible. Hence, $x_0$ belongs to $S_1(\gamma_0,\lambda_0) \subseteq U$, which is again a contradiction, since $x_n \notin U$, for all $n$. Therefore, $S_1$ is usc at $(\gamma_0, \lambda_0)$.

Now we prove that $S_1(\gamma_0, \lambda_0)$ is compact by checking its closedness. Let $x_n \in S_1(\gamma_0, \lambda_0)$ converge to $x_0$. If $x_0 \notin S_1(\gamma_0, \lambda_0)$, there exists $y_0 \in K_2(x_0, \gamma_0)$ such that

$$\langle z_0, \psi(y_0, x_0, \lambda_0)\rangle \ngeq 0. \tag{2.2}$$

Proceeding similarly as before, we arrive at a contradiction to (2.2). Hence $x_0 \in S_1(\gamma_0, \lambda_0)$. Therefore, $S_1(\gamma_0, \lambda_0)$ is closed. The compactness of $E(\gamma_0)$ derives that of

$S_1(\gamma_0, \lambda_0)$. By the condition (ii) of Lemma 1.1, it follows that $S_1$ is closed at $(\gamma_0, \lambda_0)$. And so we complete the proof. □

The following example shows that the upper semicontinuity and the compactness of $E$ are essential.

***Example 2.1.***

Let $A = B = X = Y = \mathbb{R}$, $\Gamma = \Lambda = [0,1], \gamma_0 = 0$, $K_1, K_2 : A \times \Gamma \to 2^A$, $T : A \times \Gamma \to 2^{L(X,Y)}$ and $\psi : A \times A \times \Gamma \to A$ be defined by

$K_1(x,\gamma) = (-\gamma - 1, \gamma]$, $\psi(y,x,\gamma) = \{\gamma^2 + \gamma + 2\}$,

$T(y,\gamma) = \{\frac{1}{2^{\gamma+2}}\}$, $K_2(x,\gamma) = [0, e^{\gamma^2+1}]$,

Then, we have $E(0) = (-1, 0]$ and $E(\gamma) = (-\gamma - 1, \gamma], \forall \gamma \in (0,1]$. We show that assumptions (ii) and (iii) of Theorem 2.1 are fulfilled. But $S_1$ is neither usc nor closed at $(0,0)$. The reason is that $E$ is not usc at $0$ and $E(0)$ is not compact.

In fact,

$$S_1(\gamma,\lambda) = \begin{cases} (0,1) & \text{if } \lambda = 0, \\ (-1-\lambda, \lambda) & \textit{otherwise.} \end{cases}$$

The following example shows that the lower semicontinuity of $K_2$ in Theorem 2.1 is essential.

***Example 2.2.***

Let $A = B = X = Y = \mathbb{R}$, $\Gamma = \Lambda = [0,1]$, $\gamma_0 = 0$, $K_1 : A \times \Gamma \to 2^A$,

$K_1 : A \times \Gamma \to 2^A$, $T : A \times \Gamma \to 2^{L(X,Y)}$ and $\psi : A \times A \times \Gamma \to A$ be defined by

$$K_2(x,\gamma) = \begin{cases} \{-6, 0, 6\} & \text{if } \gamma = 0 \\ \{0, 6\} & \text{if } \gamma \neq 0. \end{cases}$$

$\psi(y, x, \gamma) = \{x + y + \gamma\}$,

$T(y,\gamma) = \{1\}$

$K_1(x,\gamma) = [0,6]$.

Then $E(\gamma) = [0, 6], \forall \gamma \in [0,1]$. Hence E is usc at 0 and E(0) is compact, assumption

(iii) is satisfied. We have

$$S_1(\gamma,\lambda)=\begin{cases}\{6\} & \text{if } \gamma=0\\ [0,6] & \text{if } \gamma\in(0,1].\end{cases}$$

Therefore, $S_1$ is not usc at $(0,0)$. The reason is that $K_2$ is not lsc at $(x,0)$.

The following example shows that the all assumptions of Theorem 2.1 are satisfied.

***Example 2.3.***

Let $X=Y=\mathbb{R}$, $A=B=[0,3], \Gamma=\Lambda=[0,\ 1], \gamma_0=0$, $K_1,K_2: A\times\Gamma\to 2^A$, and $\psi: A\times A\times\Gamma\to A$ be defined by

$$K_1(x,\gamma)=K_2(x,\gamma)=[0,1],$$

$$\psi(y,x,\gamma)=\{\gamma^2+\gamma\},$$

$$T(y,\ \gamma)=\left\{\frac{1}{e^{cos^4\gamma}+sin^2\gamma+2}\right\}.$$

We see that the all assumptions of Theorem 2.1 are satisfied. So, $S_1$ is both usc and closed at $(0,0)$. In fact, $S_1(\gamma\ \lambda)=[0\ 1], \forall\gamma\in[0,1]$.

***Theorem 2.2.***

*Assume for the problem $(MQIP_2)$ that*

*(i) E is usc at $\gamma_0$ and $E(\gamma_0)$ compact set;*

*(ii) in $K_1(A,\Gamma)\times\{\gamma_0\}, K_2$ is lsc;*

*(iii) in $K_2(K_1(A,\Gamma),\Gamma)\times\{\gamma_0\}, T$ is lsc.*

*Then, $S_2$ is usc at $(\gamma_0,\lambda_0)$. Moreover, $S_2(\gamma_0,\lambda_0)$ is compact and $S_2$ is closed at $(\gamma_0,\lambda_0)$.*

***Proof.*** We omit the proof since the technique is similar as that for Theorem 2.1 with suitable modifications. □

***Remark 2.1.***

(i) In the special case, if let $\Lambda=\Gamma$ and $\psi(y,x,\gamma)=y-x, K_1(x,\gamma)=$ $K(x,\gamma)\cap A, K_2(x,\gamma)=K(x,\gamma)$ with $K: A\times\Gamma\to 2^A$. Then, the problems $(MQIP_2)$ becomes $(MVI(\gamma))$ is studied in [6].

(ii) In cases as above. Then, Theorem 3.1 in [6] is a particular case of Theorem 2.2. Moreover , the following example 2.4 shows a case where the assumed compactness in Theorems 3.1 and 3.2 of [6] is violated but the assumptions of Theorem 2.2 are ulfilled.

***Example 2.4.***

Let $X=Y=\mathbb{R}$, $\Lambda=\Gamma=[0,\ 1], A=B=[0,3), \gamma_0=0, K_1=K_2=K: A\times\Gamma\to 2^A$, $T: A\times\Gamma\to 2^{L(X,Y)}$ and $\psi: A\times A\times\Gamma\to A$ be defined by

$K_1(x,\gamma)=K_2(x,\gamma)=K(x,\gamma)=\left[\frac{1}{2},\frac{3}{2}\right],$

$\psi(y,x,\gamma) = \{x-y\},$

$T(y,\gamma)=\{1\}$ .

We show that the assumptions of Theorem 2.2 are easily seen to be fulfilled and so $S_2$ is usc and closed at $(0,0)$. However, Theorems 3.1 and 3.2 in [6] does not work. The reason is that A is not compact. In fact, $S_2(\gamma,\lambda)=\{3\}, \forall\gamma\in[0,1]$.

## 3. Lower semicontinuity of solution sets

In this section, we discuss the lower semicontinuity and the Hausdorff lower semicontinuity of the exact solution for the problems (MQIP$_1$) and (MQIP$_2$).

***Theorem 3.1.***

*Assume for the problem (MQIP$_1$) that*

*(i) $E$ is lsc at $\gamma_0$, $K_2$ is usc and compact-valued in $K_1(A,\Gamma)\times\{\gamma_0\}$;*

*(ii) in $K_2(K_1(A,\Gamma),\Gamma)\times\{\gamma_0\}$, $T$ is lsc.*

*Then $S_1$ is lsc at $(\gamma_0,\lambda_0)$.*

***Proof.***

Suppose to the contrary the existences of $x_0\in S_1(\gamma_0,\lambda_0)$ and net $\{(\gamma_n,\lambda_n)\}$ converging to $(\gamma_0,\lambda_0)$ such that, for all $x_n\in S_1(\gamma_n,\lambda_n)$, the net $\{x_n\}$ does not converge to $x_0$. Since (i), there is $x'_n\in E(\gamma_n)$, $x'_n\to x_0$. By the above contradiction assumption, there must be a subnet $\{x'_k\}$ of $\{x'_n\}$ such that $x'_k\notin S_1(\gamma_k,\lambda_k)$, for all $k$, i.e., $\exists y_k\in K_2(x'_k,\gamma_k), \exists z_k\in T(y_k,\gamma_k)$

(ii) $\langle z_k,\psi(y_k,x'_k,\lambda_k)\rangle<0.$ (3.1)

By the upper semicontinuity and the compactness of $K_2$ and $T$, there exists $y_0\in K_2(x_0,\gamma_0)$ and $z_0\in T(y_0,\gamma_0)$ such that $y_k\to y_0$ and $z_k\to z_0$ (can take subnets if necessary). By the continuity of $\psi$ and $\langle .,.\rangle$, and since (3.1) we have

$\langle z_0,\psi(y_0,x_0,\lambda_0)\rangle<0,$

which is impossible since $x_0\in S_1(\gamma_0,\lambda_0)$. Therefore, $S_1$ is lsc at $(\gamma_0,\lambda_0)$. □

The following example shows that the lower semicontinuity of $E$ is essential.

***Example 3.1.***

Let $X=Y=\mathbb{R}$, $A=B=[0,1]$, $\Gamma=\Lambda=[0,1]$, $\gamma_0=0$, $K_1,K_2: A\times\Gamma\to 2^A$, $T: A\times\Gamma\to 2^{L(X,Y)}$ and $\psi: A\times A\times\Lambda\to A$ be defined by

$$K_1(x,\gamma)=\begin{cases}\left\{-\frac{1}{3},0,\frac{1}{3}\right\} & \text{if } \gamma=0\\ \left\{0,\frac{1}{3}\right\} & \text{if } \gamma\neq 0.\end{cases}$$

$\psi(y,x,\lambda)=\gamma+\sin^4(\gamma)+\sin^2(\gamma),$

$T(y,\gamma)=\{5^{\gamma^2+3}\},$

$K_2(x,\gamma)=[0,\frac{1}{3}].$

Then, we shows that $K_2$ is usc and compact-valued in $A\times\{\gamma_0\}$ and the assumptions (ii) and (iii) of Theorem 3.1 are fulfilled. But $S_1$ is not lsc at $(0,0)$. The reason is that $E$ is not lsc at $0$. In fact,

$$S_1(\lambda,\gamma)=\begin{cases}\left\{0,\frac{1}{3}\right\} & \text{if } \gamma\in(0,1]\\ \left\{-\frac{1}{3},0,\frac{1}{3}\right\} & \text{if } \gamma=0.\end{cases}$$

The following example shows that the all assumptions of Theorem 3.1 are satisfied.

***Example 3.2.***

Let $A=B=X=Y=□$, $\Gamma=\Lambda=[0,1], \gamma_0=0$, $K_1,K_2:A\times\Gamma\to 2^A$, $T:A\times\Gamma\to 2^{L(X,Y)}$ and $\psi:A\times A\times\Lambda\to A$ be defined by

$$K_1(x,\gamma)=\begin{cases}[0,\frac{1}{2}] & \text{if } \gamma=0\\ [-\frac{1}{2},\frac{2}{2}] & \text{if } \gamma\neq 0.\end{cases}$$

$\psi(y,x,\gamma)=\{\gamma+\sin^4(\gamma)+\cos^2(\gamma)\},$

$T(y,\gamma)=\{\frac{1}{2^{\gamma^2+2}}\},$

$K_2(x,\gamma)=[0,1].$

We have $E(\gamma)=[-1,2]$ for all $\gamma\in(0,1]$ and $E(0)=[0,1]$. It is not hard to see that (i)-(iii) in Theorem 3.1 are satisfied and, according to Theorem 3.1, $S_1$ is lsc at $(0,0)$.

In fact, $S_1(\gamma,\lambda)=[-\frac{1}{2},\frac{2}{3}]$ for all $\gamma\in(0,1]$ and $S_1(0,0)=[0,\frac{1}{2}]$).

***Theorem 3.2.*** *Assume for the problem (MQIP $_2$) that*

*(i) $E$ is lsc at $\gamma_0$, $K_2$ is usc and compact-valued in $K_1(A,\Gamma)\times\{\gamma_0\}$;*

*(ii) in $K_2(K_1(A,\Gamma),\Gamma)\times\{\gamma_0\}$, $T$ is usc and compact-valued.*

*Then $S_2$ is lsc at $(\gamma_0, \lambda_0)$.*

***Proof.***

We omit the proof since the technique is similar as that for Theorem 3.1 with suitable modifications. □

Next, we study the Hausdorff lower semicontinuity of the exact solution sets for the problems (MQIP$_1$) and (MQIP$_2$).

***Theorem 3.3.***

*Impose the assumption of Theorem 3.1 and the following additional conditions:*

*(iii) $K_2(.,\gamma_0)$ is lsc in $K_1(A,\Gamma)$ and $E(\gamma_0)$ is compact;*

*(iv) in $K_2(K_1(A,\Gamma),\Gamma)$, $T(.,\gamma_0)$ is usc and compact-valued.*

*Then $S_1$ is H-lsc at $(\gamma_0, \lambda_0)$.*

***Proof.***

We first prove that $S_2(\gamma_0,\lambda_0)$ is closed. Suppose to the contrary the existence of $x_n \in S_2(\gamma_0,\lambda_0)$, $x_n \to x_0$, such that $x_0 \notin S_2(\gamma_0,\lambda_0)$, $\exists y_0 \in K_2(x_0,\gamma_0)$ and $\forall z_0 \in T(y_0,\gamma_0)$ such that

$$\langle z_0, \psi(y_0,x_0,\lambda_0)\rangle < 0, \tag{3.2}$$

By the lower semicontinuity of $K_2(.,\gamma_0)$ and $T(.,\gamma_0)$ is lsc at $x_0$ and $y_0$, there exists $y_n \in K_2(x_n,\gamma_n)$ and $z_n \in T(y_n,\gamma_n)$ such that $y_n \to y_0$ and $z_n \to z_0$. As $x_n \in S_2(\gamma_0,\lambda_0)$, then we have

$$\langle z_n, \psi(y_n,x_n,\lambda_0)\rangle \text{ Ž } 0, \tag{3.3}$$

By the continuity of $\psi$ and $\langle .,.\rangle$. So, since (3.3) yields that

$$\langle z_0, \psi(y_0,x_0,\lambda_0)\rangle \text{ Ž } 0, \tag{3.4}$$

we see a contradiction between (3.2) and (3.4). Thus, $S_1(\gamma_0,\lambda_0)$ is closed, and hence it is compact. Theorem 3.1 implies the lower semicontinuity of $S_1$. The Hausdoff lower semicontinuity of $S_1$ is direct from condition (ii) of Lemma 1.1. □

The following shows that the compactness of $E$ in Theorem 3.3 is essential.

***Example 3.3.***

Let $A = B = X = \mathbb{R}^2, Y = \mathbb{R}$, $\Gamma = \Lambda = [0,1]$, $\gamma_0 = 0$, $K_1, K_2 : A\times\Gamma \to 2^A$, $T : A\times\Gamma \to 2^{L(X,Y)}$ and $\psi : A\times A\times\Lambda \to A$ be defined by

$K_1(x,\gamma) = K_2(x,\gamma) = \{(x_1, \lambda x_1^6)\}, x = (x_1,x_2) \in \mathbb{R}^2,$

$\psi(y,x,\gamma) = \{2\gamma^6 + \cos^2(\gamma)\},$

$T(y,\gamma) = \{2^{1+\gamma^6+\sin^4(\gamma)}\}.$

We have $E(0) = \{x \in \mathbb{R}^2 \mid x_2 = 0\}$ and $E(\gamma) = \{x \in \mathbb{R}^2 \mid x_2 = \gamma x_1^4)\}, \forall\gamma \in (0,1]$.

We shows that the all assumptions of Theorem 3.3 are satisfied, but the compactness of $E(0)$ is not satisfied. Direct computations give $S_1(0,0)=\{(x_1,x_2)\in \mathbb{R}^2 \mid x_2=0\}$ and $S_1(\gamma,\lambda)=\{x\in R^2 \mid x_2=\gamma x_1^6)\}, \forall\gamma\in(0,1]$ is not Hausdorff lower semicontinuous at $(0,0)$.

***Theorem 3.4.***

*Impose the assumption of Theorem 3.2 and the following additional conditions:*

*(iii)* $K_2(.,\gamma_0)$ *is lsc in* $K_1(A,\Gamma)$ *and* $E(\gamma_0)$ *is compact;*

*(iv) in* $K_2(K_1(A,\Gamma),\Gamma)$, $T(.,\gamma_0)$ *is lsc.*

*Then* $S_1$ *is H-lsc at* $(\gamma_0,\lambda_0)$.

***Remark 3.1.***

Theorem 3.2 extends Theorem 4.1 in [6], Theorem 3.4 extends Corollary 4.1 in [6].

***Theorem 3.5.***

*Suppose that all conditions in Theorems 2.1 and 3.1 (Theorems 3.3, respectively) are satisfied. Then, we have* $S_1$ *is both continuous (H-continuous, respectively) and closed at* $(\gamma_0,\lambda_0)$.

***Theorem 3.6.***

*Suppose that all conditions in Theorems 2.2 and 3.2 (Theorems 3.4, respectively) are satisfied. Then, we have* $S_2$ *is both continuous (H-continuous, respectively) and closed at* $(\gamma_0,\lambda_0)$.

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